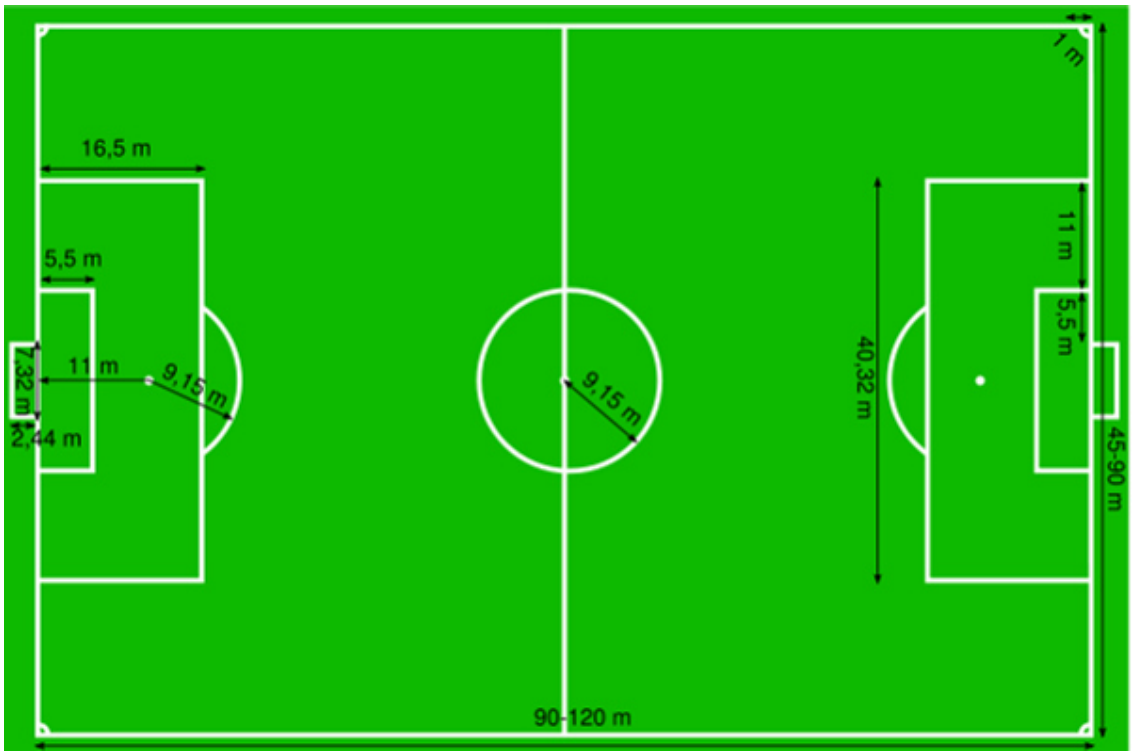
13 AREAS AND VOLUMES

FOOTBALL PITCH

The average surface area of a football pitch is 100m x 60m. For instance, the Santiago Bernabeú is 105m x 68m which is the identical dimension for the Camp Nou. That means it is a similar size to a hectare which is the surface area of 100m x 100m a square, that is 10000 square metres (m^2).

The picture shows the possible dimensions of a football pitch.

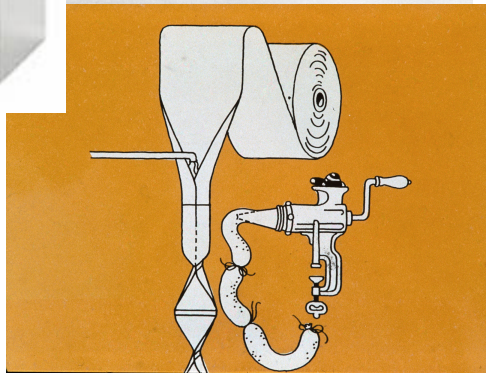


What is the total length of the white lines?

TETRA PAK

In 1943, a Swedish company lab started to work on developing the milk carton package and in 1944, Erik Wallenberg, a young lab assistant was the head of the research lab. He had the novel idea of constructing a tetrahedron shaped package from a tube of paper. The idea was simple but efficient, making optimal use of the material involved. After some initial hesitation, the director understood the potential of the package and patented it on the 27th March 1944. The rest of the 1940s were spent developing possible packaging materials and solving the technical questions of filling, sealing and distribution.

He had the idea of continuously sealing the packages of milk while filling the tube in the manner of stuffing sausages. This is a way to avoid oxygen entering the package.



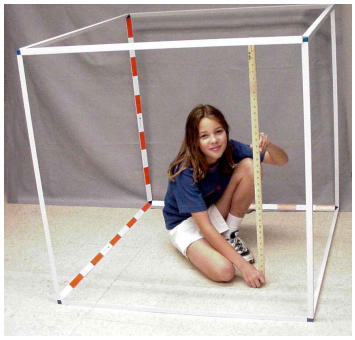
1. Write down the dimensions of a tetra brick: length, width and height.

2. Write down its capacity using the information from the cover.

3. Calculate the volume according to the above measurements.

4. What conclusion do you get?

1. UNITS OF VOLUME



VOLUME

It is the quantity of space enclosed by a solid.

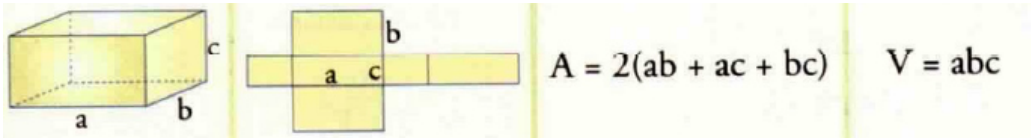
UNITS

The fundamental unit of volume is the cubic meter.
That is the volume of a cube whose edges are 1 m long.

2. AREA AND VOLUME FOR THE CUBOID, THE PRISM AND THE CYLINDER

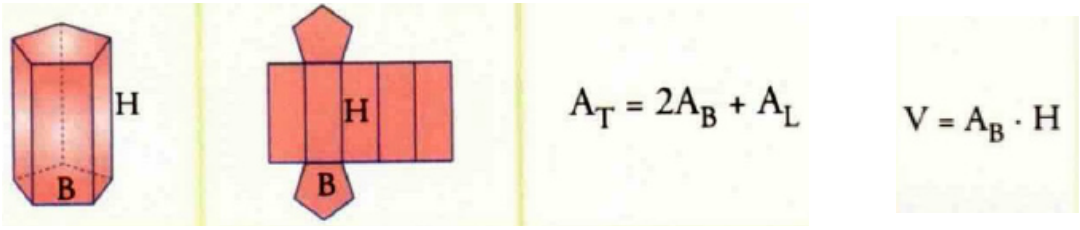
CUBOID

The cuboid area is the sum of its six faces.
The cuboid volume is the result of multiplying the three dimensions.



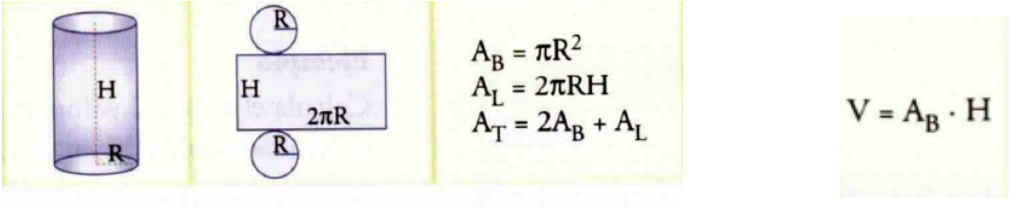
PRISM

The area of a prism is the result of adding up the areas of the faces.
The volume is the result of multiplying the base area by the height.



CYLINDER

The cylinder area is the result of adding up the two base areas and the rectangle that forms the lateral face.
The volume is the result of multiplying the base area by the

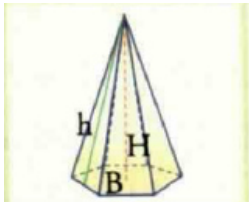


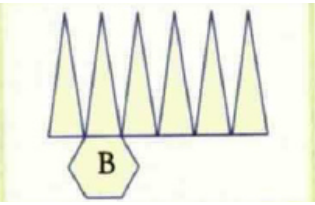
3. AREA AND VOLUME FOR THE PYRAMID, THE CONE AND THE SPHERE

PYRAMID

The total area for the pyramid is the base area plus the lateral area.

The volume is obtained by multiplying one third by the area base by the height.





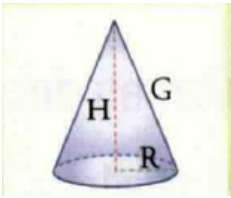
$$A_T = A_B + A_L$$

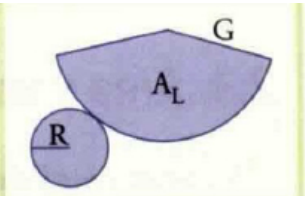
$$V = \frac{1}{3} A_B \cdot H$$

CONE

The total area for the cone is the base area –a circle- plus the lateral area –a circular sector- or circular triangle

The volume is obtained by multiplying one third by the area base by the height.





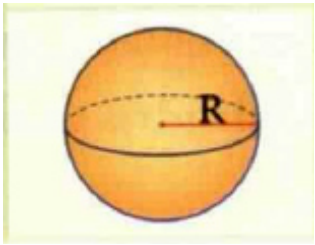
$$A_B = \pi R^2$$
$$A_L = \pi RG$$
$$A_T = A_B + A_L$$

$$V = \frac{1}{3} A_B \cdot H$$

SPHERE

The sphere is not flattened on a plane surface. However, we know the formula for this area.

The volume is obtained by multiplying four thirds by π by cubed radius.



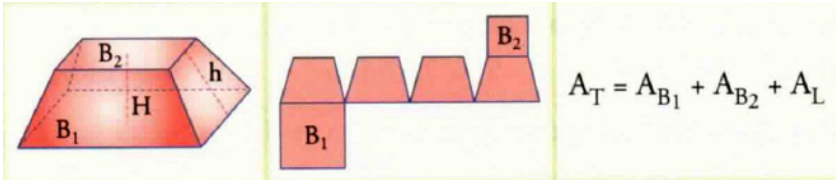
$$A = 4\pi R^2$$

$$V = \frac{4}{3} \pi R^3$$

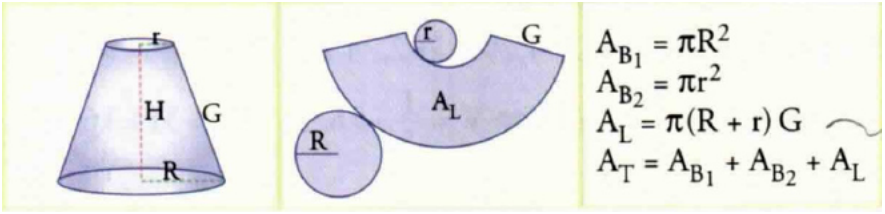
4. AREA AND VOLUME FOR A FRUSTUM

The total area for a frustum is deduced from the unfolding. It is the sum of the two bases plus the lateral area.

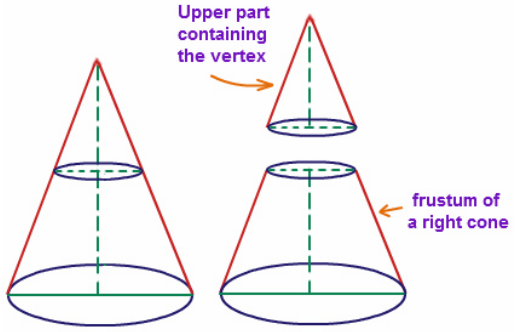
In a pyramidal frustum the lateral area is formed by trapeziums.



In a conical frustum the lateral area is a circular trapezium.



The volume could be deduced by taking away the volume of the virtual cone formed from the smallest base to the virtual apex, from the virtual cone formed by the larger base to the virtual apex.



It is similar for the pyramid frustum.

PROBLEMS AND EXERCISES

1. UNITS OF VOLUME

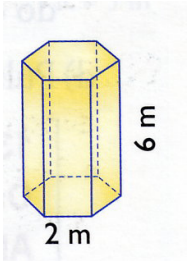
2. AREA AND VOLUME FOR THE CUBOID, THE PRISM AND THE CYLINDER

1. This is a hexagonal prism whose height is 6 m and the base edge is 2m.

a. Draw the unfolding and calculate the lateral area.

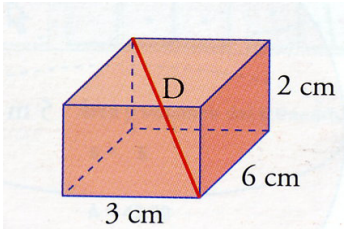
b. Calculate the base apothem.

c. Calculate the base area.

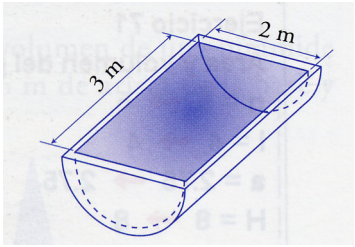


2. Work out the diagonal of the cuboid knowing that the length is 3 cm, the width is 6 cm and the height is 2 cm.

Calculate the volume and the surface area.



3. Calculate the surface area of this container. That is, the quantity of sheet iron you would need to make it. What is the volume?



4. Calculate the diagonal, lateral area, surface area and volume of a **cube** with an edge of 5 cm.

5. A swimming pool is 8 m long, 6 m wide and 1.5 m deep. The water resistant paint needed for the pool costs $6 per square meter.

a: How much will it cost to paint the interior surfaces of the pool?

b. How many liters of water will be needed to fill it?

6. How many square tiles (20 cm x 20 cm) are needed to cover the sides and base of a pool which is 10 m long, 6 meters wide and 3 m deep?

7. A moving company is trying to store boxes in a warehouse with a length of 5 m, width of 3 m and height of 2 m. How many boxes can fit in this space if they are 10 cm long, 6 cm wide and 4 cm high?

8. Four **cubes** of ice with an edge 4 cm each are left to melt in a cylindrical glass with a radius of 6 cm. How high will the water rise when they have melted?

9. Calculate the height of a **prism** whose base area is 12 dm^2 and whose capacity is 48 litres.

10. A cylindrical container with a radius of 10 cm and a height of 5 cm is filled with water. If the total mass of the filled container is 2 kg, what is the mass of the empty container?

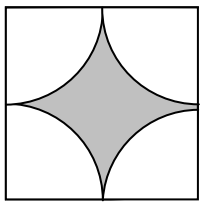
11. Calculate the volume (in cubic centimeters) of a **cuboid** that is 5 m long, 40 cm wide and 2500 mm high.

12. The height of a **cylinder** is the same length as the **circumference** of its base. Its measured height is 125.66 cm. Calculate the surface area and volume of the cylinder.

13. The side of a regular octagon measures 10 cm and its apothem is 12 cm.
a. Draw the octagon and its data.
b. Calculate the area.
c. Calculate the radius of the polygon.

14. The two equal sides of an isosceles triangle measure 50 cm and the height is 38 cm.
a. Draw the triangle with its data.
b. Find the length of its base –the unequal side-.
c. Calculate the area.

15. Calculate the shaded area in the figure knowing that the side of the square is 10 m.

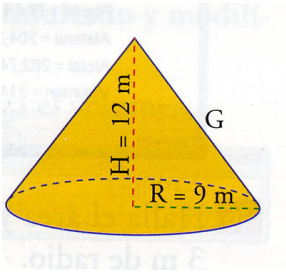


16. a. Calculate the quantity of sheet metal you would need to make 10 cylindrical containers with a diameter of 10 cm and a height of 20 cm.
b. What is the each volume?

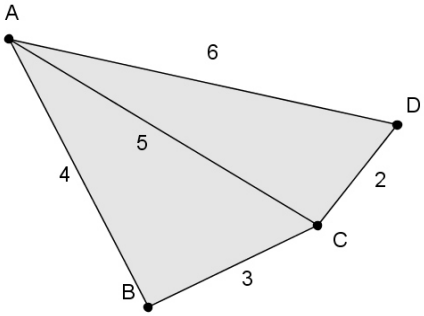
17. Calculate the lateral area, surface area and volume of a **prism** whose height is 10 cm and its base is a **rhombus** with diagonals of 12 and 18 cm in length.

3. AREA AND VOLUME FOR THE PYRAMID, THE CONE AND THE SPHERE

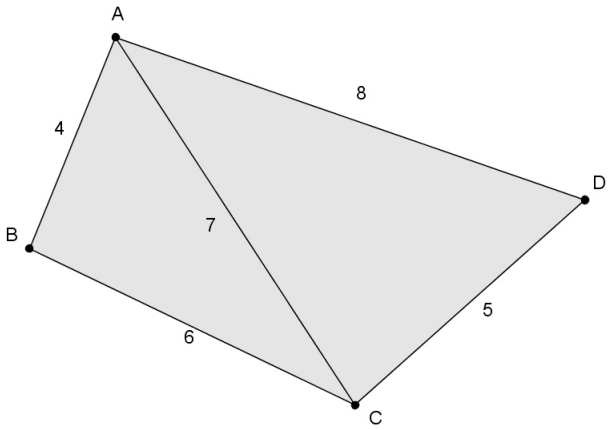
18. Calculate the slant height length (G) for the next cone knowing that the height is 12 m (H) and the radius of the base is 9m (R). Calculate the volume and the surface.



19. Calculate the surface area for the drawing quadrilateral knowing the measures you see on the picture, that is, the sides and the diagonal.



20. Calculate the surface area for the drawing quadrilateral knowing the measures you see on the picture, that is, the sides and the diagonal.



21. Work out the volume for a quadrangular pyramid whose base edge is 3m and its height is 6m. Give your result to two decimal places.

22. Calculate the lateral area, surface area and volume of a square pyramid whose base edge is 10 cm and its height is 12 cm.

23. Calculate the lateral area, surface area and volume of a hexagonal pyramid with a base edge of 16 cm and a side edge of 28 cm.

24. Calculate the lateral area, surface area and volume of a **cone** whose **slant height** is 13 cm and base radius is 5 cm.

25. Calculate the lateral area, surface area and volume of a cone whose height is 4 cm and base radius is 3 cm.

26. A quadrangular pyramid has 3 m base edge and 6 m height.

a) Draw the pyramid and write down the measures on it.

b) Calculate the base apothem.

c) Calculate the pyramid apothem.

d) Calculate the total surface.

27. Calculate the surface area and the volume of a **regular tetrahedron** with an edge of 5 cm.

28. For a party, Louis has made about 10 conical hats out of cardboard. How much cardboard was used in total if each cap has a radius of 15 cm and a slant height of 25 cm?

29. The dome of a cathedral has a semispherical form with a radius of 50 m. If the restoration costs \$300 per m^2, what is the total cost of the restoration?

30. A cube with an edge of 20 cm is filled with water. Would this amount of water fit in a **sphere** with a 20 cm radius?

4. AREA AND VOLUME FOR A FRUSTUM

31. Calculate the lateral area, surface area and volume of a **truncated cone** with radii of 2 and 6 cm and a height of 10 cm.

32. Calculate the lateral area, surface area and volume of a truncated cone with radii of 12 and 10 cm and a **slant height** of 15 cm.

33. The Earth's radius is about 6000 km; what is the volume?